MCA 3rd Semester Examination 2013
Paper - IX
(Discrete Mathematics)

Time : 3 Hrs. Full Marks : 80

1. Answer any five questions from the following $2\times5 = 10$

(a) If A and B are any finite sets, then show that $A - B = A \cap B'$, where B' is the complement of B.

(b) Construct the truth table for
$(p \rightarrow q) \rightarrow (p \wedge q)$
where p and q are propositions.

(c) Prove that the set of fourth roots of unity is an Abelian group under multiplication of complex numbers.

(d) What are the properties of an equivalence relation?

(e) Explain the rule of sum and the rule of product.

(f) Show that the number of vertices of odd degree in a graph is an even number.

(g) Show that
$f \circ g \neq g \circ f$,
Where $f(x) = x^2 + 1$ and $g(x) = \sin x$.

2. Answer any three questions from the following $4\times3 = 12$

(a) If a sequence is defined recursively by
$a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ and
$a_1 = a_2 = 1$,
find a_{12}.

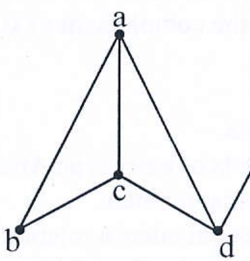
(b) Using truth table, show that
(i) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(ii) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
where p and q are propositions.

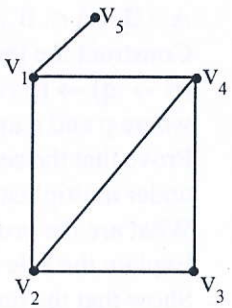
(c) A foot ball stadium has five gates on the eastern boundary and four gates on the western boundary.
(i) In how many ways can a person enter through an east gate and leave by a western gate?
(ii) In how many different ways in all can a person enter and get out through different gates?

(d) Show that the relation 'being a subset of' is a partially ordered set on the power set of a non empty set.

(e) Prove that the following graphs G and H are isomorphic.



G (V, E)



H (V′, E′)

3. Answer any three of the following $6 \times 3 = 18$

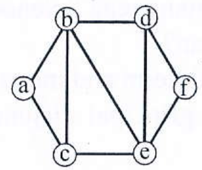
(a) Define distributive lattice.
Show that a lattice L is distributive if and only if
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
for all a, b, c ∈ L.

(b) How many arrangements can be made with the letters of the word 'MATHEMATICS' ?
In how many of them vowels are together?

(c) Let (B, +, ·, ′, 0, 1) be a Boolean Algebra, then show that for all a, b ∈ B
(i) a + a = a, a.a = a

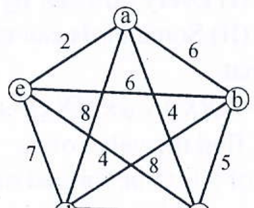
(ii) a + 1 = 1, a.0 = 0
(iii) a + (a.b) = a, a.(a + b) = a

(d) Construct spanning trees of the following graph G, using Depth-First search and Breadth-First search Algorithms.



G (V, E)

(e) Describe Prim's Algorithm to find a minimum spanning tree. Using this Algorithm, find a minimum spanning tree for the following connected weighted graph G.



G (V, E)

4. Answer any four of the following $10 \times 4 = 40$

(a) Describe 'Addition modulo n' and 'multiplication moduleo n'. Show that (z_n, t_n) is an Abelian group where $z_n = \{0, 1, 2.... n-1\}$

(b) In a survey of 100 families, the numbers that read the most recent issues of various magazines were found to be as follows :

Reader Digest	28
Reader Digest and Science Today	8
Science Today	30
Reader Digest and Caravan	10
Caravan	42

Science Today and Caravan 5
All the three Magazines 3
using set theory, find

(i) How many read none of the three magazines?

(ii) How many read Caravan as their only magazine?

(iii) How many read Science Today if and only if they read Caravan?

(c) (i) Define minterm and max term for a number of variables.

(ii) What is principal disjunctive normal form of a given formula.

(iii) Find the method to obtain the principal disjunctive normal form of a given formula without constructing its truth table.

(d) (i) Express the following statements using quantifiers. Then construct the negation of the statement Express the negation in simple English.

(I) Every bird can fly

(II) Some birds can talk

(ii) Prove that

$(\forall x (P(x) \rightarrow Q(x)) \wedge \forall x (Q(x) \rightarrow S(x)) \Rightarrow \forall x(P(x) \rightarrow S(x))$

(e) If ${}^{n}c_{12} = {}^{n}c_{5}$, find the value of n.

A comittee of 5 is to be formed out of 7 gents and 4 ladies. In how many ways this can be done when

(i) at least 2 ladies are included

(ii) at most 2 ladies are included

(f) (i) State Kuratowski theorem of planar graph

(ii) Show that the graphs $K_{3,2}$ and K_5 are non planar.

(iii) If G is a connected simple planar graph without loops and has n vertices,

$e \geq 2$ edges and r regions, then show that

$$\frac{3r}{2} \leq e \leq 3n - 6.$$