

Form No. Ex- 8

Sl. No..... 060

Particulars about the candidate and the subject are **checked thoroughly** and corrected where necessary.



CENTRE CODE :.....

Invigilator

Signature of Officer-in-Charge

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

Degree 4th Sem. Examination, 2014

Integral Calculus and Differential Equations

Paper - 03

Time : 3 Hrs. Full Marks : 80

Enrolment Number

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Medium of Answer :

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INSTRUCTIONS TO CANDIDATES

1. This booklet contains...24... Pages numbering...23...Please verify number of pages in the booklet before answering.
2. An Examinee is allowed to bring only Admission Card and Identity Card to the Examination Hall. Any Examinee found in possession of loose papers, books etc. is liable to be Expelled.
3. Enrolment No. and Medium of answer must be written legibly at the specified places. Examinee's name and any other identifying mark which reveals examinees identity shall not be written anywhere in the script.
4. For Making calculations, only the last page provided for rough work shall be used.
5. No pages of the script be torn out .
6. Calculators will not be allowed for making calculations in the examination hall. **MOBILE PHONES are strictly prohibited in the examination Centre.**
7. No candidate will be allowed to leave or go out of the hall during the First hour of the Examination.
8. A candidate having completed his/her answer, the script must be handed over, to an invigilator before leaving the hall.
9. Contravention of any of the instructions mentioned above shall render a candidate liable for disciplinary action as per regulations of the University.

292(BMAP-03)

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Que. No.	Marks
1.	
2.	
3. a	
3. b	
3. c	
3. d	
3. e	
3. f	
3. g	
4. a	
4. b	
4. c	
5. a	
5. b	
5. c	
Total	

1. Answer any eight from the following questions

1×8 = 8

তলৰ প্রশ্নসমূহৰ যিকোনো আঠটাৰ উত্তৰ লিখক

(a) Find the value of :

মান নির্ণয় কৰক :

$$\int \frac{1}{\sqrt{4-x^2}} dx$$

(b) If $f(x)$ is an odd function, then write down the value of $\int_{-a}^a f(x) dx$

$f(x)$ এটা অযুগ্ম ফলন হ'লে, $\int_{-a}^a f(x) dx$ ৰ মান লিখক।

(c) Write down the order of the following differential equation :

তলৰ অৱকল সমীকৰণটোৰ ক্ৰমটো লিখক :

$$\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^5 + 3y = 0$$

(d) Write down the degree of the following differential equation :

তলৰ অৱকল সমীকৰণটোৰ মাত্ৰাটো লিখক :

$$y^2\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} = 4\sin x$$

(e) What is the necessary and sufficient condition for $M(x,y) dx + N(x,y) dy = 0$ to be an exact equation?

$M(x,y) dx + N(x,y) dy = 0$ যথার্থ হোৱাৰ আৱশ্যকীয় আৰু পৰ্যাপ্ত চৰ্তটো কি?

(f) Write down the solution of the following equation :

তলৰ সমীকৰণটোৰ সমাধানটো লিখক :

$$y = px + \frac{4}{p^2}, p = \frac{dy}{dx}$$

(g) Write down the complementary function of the ordinary linear differential equation.

$$\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2y = X, \text{ if its auxiliary equation has real and distinct roots } m_1 \text{ and } m_2.$$

সাধাৰণ ৰৈখিক অৱকল সমীকৰণ $\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2y = X$, ৰ সহায়ক সমীকৰণটোৰ বাস্তব আৰু ভিন্ন

মূল দুটা m_1 আৰু m_2 হ'লে, সমীকৰণটোৰ পূৰক ফলনটো লিখক।

(h) Express the arc length S of the curve $y = f(x)$ between two points for which $x = a$ and $x = b$, $b > a$ in the form of a definite integral.

$y = f(x)$ বক্ৰৰ $x = a$ আৰু $x = b$ ৰ মাজৰ চাপৰ দৈৰ্ঘ্য S অক এটা নিশ্চিত অনুকলনৰ ৰূপত প্ৰকাশ কৰক। ইয়াত $b > a$ ।

(i) Write down the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ উপবৃত্তটোৰ কালিৰ মান লিখক।}$$

(j) Write down whether the following differential equation is linear or non-linear :

নিম্ন প্ৰদত্ত অৱকল সমীকৰণটো ৰৈখিক নে অৰৈখিক লিখক :

$$x^2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xe^x$$

2. Answer any eight from the following questions

2×8 = 16

তলৰ প্ৰশ্নসমূহৰ যিকোনো আঠটাৰ উত্তৰ লিখক

(a) Evaluate :

মান নিৰ্ণয় কৰক :

$$\int x\sqrt{x^2 + 5} dx$$

(b) Evaluate :

মান নিৰ্ণয় কৰক :

$$\int xe^{-2x} dx$$

(c) Show that :

দেখুৱাওক যে :

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

(d) Evaluate :

মান নিৰ্ণয় কৰক :

$$\int_0^{\sqrt{\frac{\pi}{2}}} x \cos x^2 dx$$

(e) Find by integration the area of the circle $x^2 + y^2 = 16$

অনুকলনৰ সহায়ত $x^2 + y^2 = 16$ বৃত্তটোৰ কালি উলিয়াওক।

(f) Eliminating a and b, form a differential equation from

$$x = a \cos nt + b \sin nt$$

$x = a \cos nt + b \sin nt$ ৰ পৰা a আৰু b অপনয়ন কৰি এটা অৱকল সমীকৰণ গঠন কৰক।

(g) Solve :

সমাধান কৰক :

$$\frac{dx}{1+x^2} - \frac{dy}{1+y^2} = 0$$

(h) Solve :

সমাধান কৰক :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6 = 0$$

(i) Solve :

সমাধান কৰক :

$$(D^2 + 16)y = 0$$

(j) Show that :

দেখুৱাওক যে :

$$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

3. Answer any five from the following questions

4×5 = 20

তলৰ প্রশ্নসমূহৰ যিকোনো পাঁচটাৰ উত্তৰ লিখক

(a) Evaluate :

মান নিৰ্ণয় কৰক :

$$\int \frac{1}{x^2 + 4x + 13} dx$$

(b) Evaluate :

মান নিৰ্ণয় কৰক :

$$\int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$$

(c) Show that :

দেখুৱাওক যে :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \pi/4$$

(d) If $I_n = \int \tan^n x dx$, then show that

$$I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}.$$

যদি $I_n = \int \tan^n x dx$, তেন্তে দেখুৱাওক যে $I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$.

(e) Verify that $\tan^{-1} y = x + y$ is a solution of the differential equation

$$y^2 \frac{dy}{dx} + y^2 + 1 = 0$$

সত্যাপন কৰক যে $y^2 \frac{dy}{dx} + y^2 + 1 = 0$ অৱকল সমীকৰণৰ এটা সমাধান হ'ল $\tan^{-1} y = x + y$.

(f) Form the partial differential equation from the following equation by eliminating a and b :

নিম্ন প্ৰদত্ত সমীকৰণটোৰ পৰা a আৰু b অপনয়ন কৰি এটা আংশিক অৱকল সমীকৰণ গঠন কৰক :

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

(g) Solve :

সমাধান কৰক :

$$y z p + x z q = xy$$

4. Answer any two from the following questions

8×2 = 16

তলৰ প্ৰশ্নসমূহৰ যিকোনো দুটাৰ উত্তৰ লিখক

(a) If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, then show that

$$I_n = \frac{n-1}{n} I_{n-2}$$

Hence, evaluate $\int_0^{\pi/2} \sin^4 x \, dx$

যদি $I_n = \int_0^{\pi/2} \sin^n x \, dx$, তেনেহ'লে দেখুৱাওক যে $I_n = \frac{n-1}{n} I_{n-2}$. ইয়াৰ সহায়ত $\int_0^{\pi/2} \sin^4 x \, dx$ ৰ

মান উলিয়াওক।

(b) Find the curved surface of the solid which is generated by the revolution about the x-axis of the area bounded by the x-axis, the parabola $y^2 = 16x$ and the ordinate $x = a$

$y^2 = 16x$ অধিবৃত্ত, কোটি $x = a$ আৰু x অক্ষই আগুৰা অংশৰ কালি x ওক্ষ সাপেক্ষে পৰিভ্ৰমণ কৰোৱাৰ ফলত সৃষ্টি হোৱা ঘনৰ বক্রপৃষ্ঠৰ কালি নিৰ্ণয় কৰক।

(c) (i) Form the differential equation of all circles with centre on the y-axis and passing through the origin.

যিবোৰ বৃত্ত মূল বিন্দুৰ মাজেৰে যায় আৰু যিবোৰৰ কেন্দ্ৰ y- অক্ষত থাকে, তেনেবোৰ বৃত্তৰ বাবে অৱকল সমীকৰণ গঠন কৰক।

(ii) Solve:

সমাধান কৰক :

$$y = px + f(p), \quad p = \frac{dy}{dx}$$

5. Answer any two from the following questions

তলৰ প্ৰশ্নসমূহৰ যিকোনো দুটাৰ উত্তৰ লিখক

(a) Solve :

সমাধান কৰক :

(i) $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

(ii) $\cos^2 x \frac{dy}{dx} + y = \tan x$

(b) Solve :

সমাধান কৰক :

$(D^3 + 3D^2 + 2D) y = x^2$

(c) Evaluate, using the method of integration as the limit of a sum :

সমষ্টি এটাৰ সীমা হিচাপে প্ৰকাশ কৰি অনুকল নিৰ্ণয় কৰা পদ্ধতিৰে মান নিৰ্ণয় কৰক :

$$\int_a^b x^2 dx$$